Can Flexibility-based Model Reduction Techniques Become in Vogue Again?

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ABSTRACT
We reexamine the age old issue of the displacement vs. flexibility method as applied to the model reduction strategies in structural dynamics first, then to coupled systems. It is shown that the distinction of the two methods become blurred, and the resulting model reduction process ends up employing both the displacement and a flexibility. The efficacy of the flexibility method is shown to be equivalent to that of the classical Craig-Bampton(CB) method. Parallelization of model reduction process favors flexibility methods for their scalability properties as the CB method is not amenable to scalable parallelization.

I. INTRODUCTION

The Craig-Bampton method [1] is perhaps the most widely used stiffness-based method for the model reduction of large-scale structural dynamics equations, in large part due to the adoption of stiffness method over the flexibility method. In recent years, flexibility-based methods [2] are being re-assessed in structural damage detection, model reduction, and system identification, albeit mechanics researchers continue to utilize the stiffness-based finite element software. This resurgence of flexibility methods is in large part benefitting from the advances in solution methods, in particular, partitioned solution of large systems, and also in part from the fact that reduced flexibility is more easily identifiable than reduced-stiffness in reduced-order models. This paper presents: a survey of recent advances in flexibility-based model reduction schemes and their applications, and the viability of flexibility-based model reduction relative to the Craig-Bampton method.

II. MODEL REDUCTION OVERVIEW

The partitioned equations of motion consisting several substructures may be written as [2]

$$\begin{bmatrix} \bar{\mathbf{K}} & \mathbf{B} & \mathbf{0} \\ \mathbf{B}^T & \mathbf{0} & -\mathbf{L}_f \\ \mathbf{0} & -\mathbf{L}_f^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \lambda_\ell \\ \mathbf{u}_f \end{Bmatrix} = \begin{Bmatrix} \mathbf{f} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

$$\bar{\mathbf{K}} = \mathbf{M}D^2 + \mathbf{C}\ D + \mathbf{K}, \quad D = \frac{d}{dt}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 \end{bmatrix}$$

where **M** is the mass matrix, **K** is the stiffness matrix, **B** is the substructure interface displacement compatibility Boolean matrix, **u** is the substructural displacement, λ is the substructural interface force, $\mathbf{u_f}$ is the interface displacement, $\mathbf{L_f}$ is the force balance along the substructural interface, and **f** is the applied force, respectively.

Model reduction refers to the reduction of orders of **M** and **K,** as well as associated degrees freedom. If λ is eliminated, the resulting model reduction is labeld the displacement method. If, however, $\mathbf{u_f}$ and **u** are eliminated, then it is called a hybrid method or flexibility method. Relative merits of these two resulting methods will be reviewed and application examples will be used to demonstrate the salient features of the two methods.

REFERENCES

1. R. R. Craig, Jr. and M. C. C. Bampton, Coupling of Substructures for Dynamic Analysis, *AIAA Journal*, 1968, v. 6, 1313-1319
2. K. C. Park and Y. H. Park, Partitioned Component Mode Synthesis via A Flexibility Approach, *AIAA Journal*, 2004, vol.42, 1236-1245.