A new stabilized finite element method for incompressible viscoelastic flows: the Variational Multiscale framework

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ABSTRACT

A new variational multiscale formulation is presented for computing non-Newtonian viscoelastic flows described by the Oldroyd-B model. This work is an extension of our earlier works on VMS formulations for the incompressible shear-rate dependent fluids to the non-Newtonian viscoelastic fluids [1]. The weak forms of the momentum, continuity and constitutive equations are cast in the VMS framework that leads to a two-level description of the problem. Consistent linearization of the fine-scale problem with respect to the fine-scale field and the use of bubble functions to expand the fine-scale trial and test functions lead to an analytical expression for the stabilization tensor.

I. INTRODUCTION

The viscoelastic flows are commonly encountered in a wide variety of applications in engineering, physiology and pharmaceutics. The viscoelasticity is developed by memory effects from the complex microstructures of the fluids. In physiology, human blood composed of red blood cells, white blood cells, platelets and the plasma, shows the viscoelastic characteristic projected by the elastic material property of red blood cells, one of the most dominant components in blood.

In order to embody the viscoelastic stress in computational fluid dynamics (CFD), various differential constitutive equations have been presented and the Oldroyd–B model is considered as one of the most popular models to minimize non-physical behaviors. Employing the Oldroyd-B model, however, causes numerical instability due to its hyperbolic constitutive equation beside the saddle point problem formed by the momentum and continuity equations. Accordingly, numerically robust stabilization framework is essential to obtain accurate results from computational simulations for viscoelastic fluids.

II. NONLINEAR STABILIZED FORM

The mixed nonlinear stabilized form for the incompressible viscoelastic fluids is presented.

$$
\begin{aligned}
&\rho\left(\bar{\boldsymbol{w}},\bar{\boldsymbol{v}}_{,t}\right)+\rho\left(\bar{\boldsymbol{w}},\boldsymbol{v}\cdot\nabla\boldsymbol{v}\right)+\left(\nabla\bar{\boldsymbol{w}},2\beta\eta\,\boldsymbol{\varepsilon}\left(\boldsymbol{v}\right)\right)\\
&-\left(\nabla\cdot\bar{\boldsymbol{w}},p\right)+\left(\nabla\bar{\boldsymbol{w}},\boldsymbol{\sigma}_p\right)+\left(q,\nabla\cdot\boldsymbol{v}\right)\\
&+\left(\boldsymbol{\psi}+\tau_\sigma\boldsymbol{v}\cdot\nabla\boldsymbol{\psi},\begin{pmatrix}\lambda\boldsymbol{\sigma}_{p,t}+\boldsymbol{\sigma}_p+\lambda\boldsymbol{v}\cdot\nabla\boldsymbol{\sigma}_p-\lambda\boldsymbol{\sigma}_p\cdot\nabla\boldsymbol{v}\\-\lambda\left(\nabla\boldsymbol{v}\right)^T\cdot\boldsymbol{\sigma}_p-2(1-\beta)\eta\,\boldsymbol{\varepsilon}\left(\boldsymbol{v}\right)\end{pmatrix}\right)\\
&-\left(\boldsymbol{\chi}_1+\boldsymbol{\chi}_2,\boldsymbol{\tau}\boldsymbol{r}\right)=\rho\left(\bar{\boldsymbol{w}},\boldsymbol{f}\right)+\left(\bar{\boldsymbol{w}},\boldsymbol{h}\right)_{\Gamma_h}
\end{aligned}
$$

where $\boldsymbol{v}$ is the velocity, p is the hydrodynamic pressure, $\boldsymbol{\sigma}_p$ is the extra stress tensor, $\boldsymbol{\chi}_1$ and $\boldsymbol{\chi}_2$ are the weighting components derived by the VMS framework, $\boldsymbol{\tau}$ and τ_σ are the stabilization tensors derived by the VMS framework, ρ is the density, $\boldsymbol{f}$ is the body force vector, $\boldsymbol{h}$ is the vector of prescribed boundary tractions, $\beta(=\eta_s/\eta)$ is ratio of solvent viscosity η_s and total viscosity η and λ is the stress relaxation time.

III. NUMERICAL VALIDATION

The present method is tested through flow past a circular cylinder in a channel that is standard benchmark problem to validate computational methods for viscoelastic fluids. For high Weissenberg numbers, the proposed method shows stable and accurate numerical results with lower computational cost than competing methods

REFERENCES

1. A. Masud, J. Kwack, “A stabilized mixed finite element method for the incompressible shear-rate dependent non-Newtonian fluids: Variational Multiscale framework and consistent linearization,” Comp. Methods Appl. Mech. Eng., Submitted, 2009.